Examples 6 in Basic Geometry

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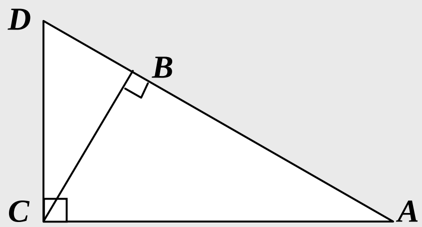
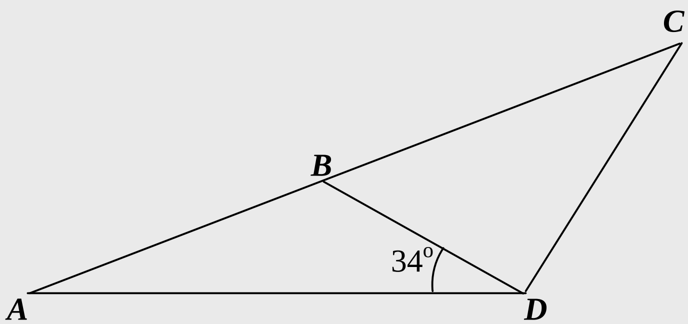
Examples 6

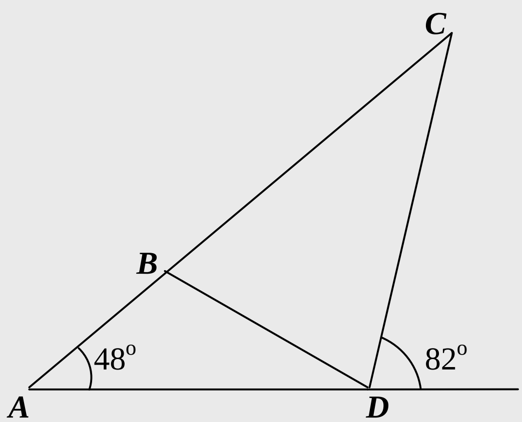
0. Assuming $AC = 5$, and $AB = 4$, find the lengths of BD and CD, and the ratio between the areas of ΔBCD, ΔBCA, and ΔCDA.



1. Assuming $\angle ABD = \angle ADC$, $BD = 3$, $AD = 8$, and $CD = 5$, find the angle C and the length of AC.



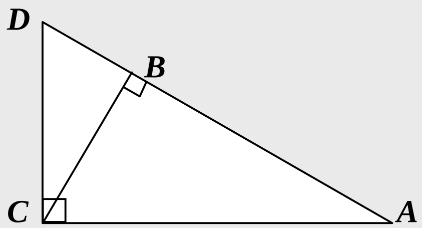
2. Assuming $\angle ADB = \angle ACD$, $AB = 3$, and $AD = 5$, find the angle C and the length of AC.



Suggestions or Solutions
To the Problem 0

Assuming in the figure below, $AC = 5$, and $AB = 4$, find the lengths of BD and CD, and the ratio between the areas of ΔBCD, ΔBCA, and ΔCDA.

Fig. 0.0



To begin with, we can see that the three triangles given are right triangles. They are not only so. But they are similar triangles, too. And if two triangles are similar, each pair of corresponding sides keeps the same ratio.

Looking at ΔBCD and ΔBCA, we can say that AB corresponds to CB, AC corresponds to CD, and BC corresponds to BD. So we get $\frac{AB}{CB} = \frac{AC}{CD} = \frac{BC}{BD}$.

How then, can we get BD and CD?

We have $AC = 5$, and $AB = 4$.

So we get $\frac{AB}{CB} = \frac{AC}{CD} = \frac{BC}{BD} \Rightarrow \frac{4}{CB} = \frac{5}{CD} = \frac{BC}{BD}$.

Thus, finding BC, and using $\frac{4}{CB} = \frac{5}{CD}$, we can get CD.

Also, using $\frac{4}{CB} = \frac{BC}{BD}$, we can get BD.

And of course, after finding CD, and using $\frac{5}{CD} = \frac{BC}{BD}$, we can get BD, too.

How then can we find BC?

In ΔBCA, the hypotenuse is AC, which is 5, AB is a leg, and is 4, and BC is the other leg. So simply using the distance formula, we can get BC.

That is, assuming $b = BC$, we get $5^2 = 4^2 + b^2 \Rightarrow b^2 = 25 - 16 = 9 = 3^2$.

And since $b = BC > 0$, we get $BC = 3$.

So next, we get $\frac{4}{BC} = \frac{5}{CD} \Rightarrow CD = \frac{3}{4} \cdot 5 = \frac{15}{4}$.

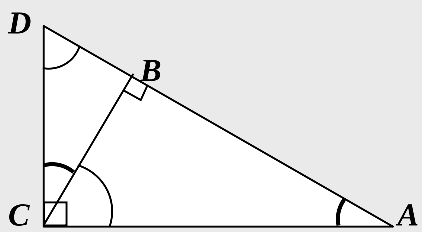
And next, we get $\frac{4}{BC} = \frac{BC}{BD} \Rightarrow BD = \frac{3}{4} \cdot 3 = \frac{9}{4}$.

How then can we find the ratio between the areas of ΔBCD, ΔBCA, and ΔCDA?

We know that for similar triangles, the ratio between the areas is the square of the ratio between the corresponding sides.
And ΔBCD, ΔBCA, and ΔCDA are similar triangles. How?

We can put angles in the figure above the way below.

Fig. 0.1



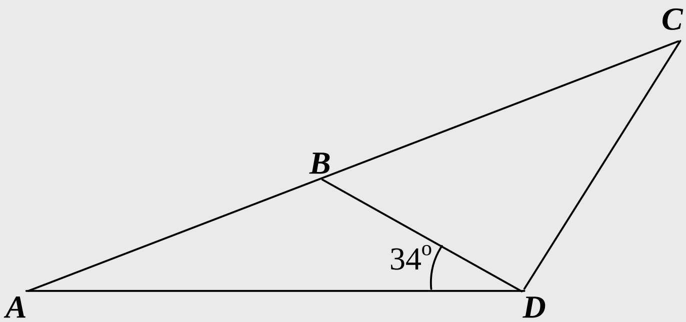
So the three triangles share the same set of three angles.

And assuming BC is in ΔCDA, BA is in ΔBCA, and CA is in ΔCDA, and the three sides BC, BA, and CA are corresponding to each other, we can set $BC : BA : CA = 3 : 4 : 5$.
So assuming p is the area of ΔCDA, q is the area of ΔBCA, and r is the area of ΔCDA, we get $p : q : r = 3^2 : 4^2 : 5^2$.

Suggestions or Solutions
To the Problem 1

Assuming in the figure below, $\angle ABD = \angle ADC$, $BD = 3$, $AD = 8$, and $CD = 5$, find the angle C and the length of AC.

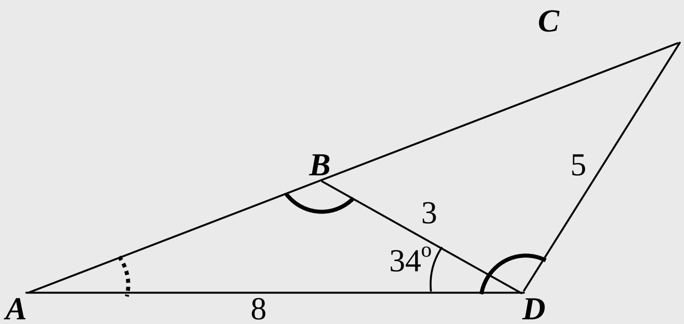
Fig. 1.0



To begin with, we can see two triangles are similar. What are the two though?

Putting in the figure above, the angles given and the sides given, we get this:

Fig. 1.1



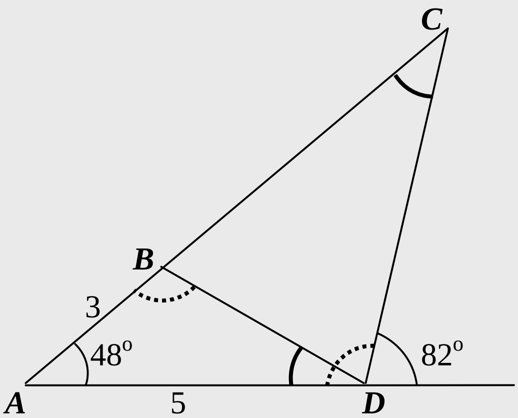
So we can say that ΔABD is similar to ΔADC.
Thus, we can see that the angle C is 34°. How then can we get the length of AC?

We know that ΔABD is similar to ΔADC.
So we can say that AD corresponds to AC, AB corresponds to AD, and BD corresponds to DC. Thus, we get $AD / AC = AB / AD = BD / DC$.
So we get $8 / AC = AB / 8 = 3/5$.
Thus, we get $8 / AC = 3/5 \Rightarrow AC = (5/3)8 = 40/3$.

Suggestions or Solutions
To the Problem 2

Assuming $\angle ADB = \angle ACD$, $AB = 3$, and $AD = 5$, find the angle C and the length of AC.

Fig. 2.0



So putting in the figure above, the angles and the sides given, we can see that ΔABD is similar to ΔADC. How then can we find the angle C?

First, we can say that $\angle ADC + 82^o = 180^o$. So we get $\angle ADC = 98^o$.

And we know $\angle ABD = \angle ADC = 98^o$, and $\angle ADB = \angle C$.

Also, we know that the sum of the three angles in a triangle is 180^o.
So next, in ΔABD, we can see that $48^o + \angle ABD + \angle ADB = 48^o + 98^o + \angle C = 180^o$.
Thus, we get $\angle C = 34^o$. How then can we get the length of AC?

We know that ΔABD is similar to ΔADC.
So we can say that AD corresponds to AC, AB corresponds to AD, and BD corresponds to DC. Thus, we get $\frac{AD}{AC} = \frac{AB}{AD} = \frac{BD}{DC}$.

And we have $\boldsymbol{AB} = \mathbf{3}$, and $\boldsymbol{AD} = \mathbf{5}$.

So we get $\frac{\boldsymbol{AD}}{\boldsymbol{AC}} = \frac{\boldsymbol{AB}}{\boldsymbol{AD}} \Rightarrow \frac{5}{\boldsymbol{AC}} = \frac{3}{5} \Rightarrow \boldsymbol{AC} = \frac{5}{3} \cdot \mathbf{5} = \frac{25}{3}$.